MUHANDISLIK & IQTISODIYOT

ijtimoiy-iqtisodiy, innovatsion texnik, fan va ta'limga oid ilmiy-amaliy jurnal

2026-YIL

AVGUST 8-SON, I-QISM

Milliy nashrlar

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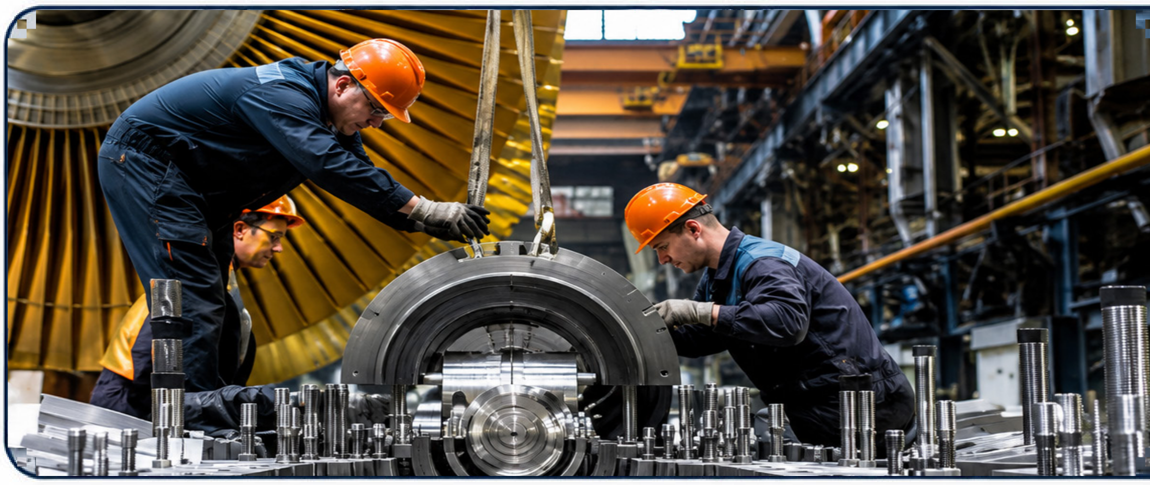
05.00.00 - Texnika fanlari

08.00.00 - Iqtisodiyot fanlar



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05.01.00 – Axborot texnologiyalari, boshqaruv va kompyuter grafikasi
05.01.01 – Muhandislik geometriyasi va kompyuter grafikasi. Audio va video texnologiyalari
05.01.02 – Tizimli tahlil, boshqaruv va axborotni qayta ishlash
05.01.03 – Informatikaning nazariy asoslari
05.01.04 – Hisoblash mashinalari, majmualari va kompyuter tarmoqlarining matematik va dasturiy ta'minoti
05.01.05 – Axborotlarni himoyalash usullari va tizimlari. Axborot xavfsizligi
05.01.06 – Hisoblash texnikasi va boshqaruv tizimlarining elementlari va qurilmalari
05.01.07 – Matematik modellashtirish
05.01.11 – Raqamli texnologiyalar va sun'iy intellekt
05.02.00 – Mashinasozlik va mashinashunoslik
05.02.08 – Yer usti majmualari va uchish apparatlari
05.03.02 – Metrologiya va metrologiya ta'minoti
05.04.01 – Telekommunikasiya va kompyuter tizimlari, telekommunikasiya tarmoqlari va qurilmalari. Axborotlarni taqsimlash
05.05.03 – Yorug'lik texnikasi. Maxsus yoritish texnologiyasi
05.05.05 – Issiqlik texnikasining nazariy asoslari
05.05.06 – Qayta tiklanadigan energiya turlari asosidagi energiya qurilmalari
05.06.01 – To'qimachilik va yengil sanoat ishlab chiqarishlari materialshunosligi
05.08.03 – Temir yo'l transportini ishlatish
05.08.06 – "G'ildirakli va gusenisali mashinalar va ularni ishlatish" (texnika fanlari)
05.09.01 – Qurilish konstruksiyalari, bino va inshootlar
05.09.04 – Suv ta'minoti. Kanalizatsiya. Suv havzalarini muhofazalovchi qurilish tizimlari
10.00.06 – Qiyosiy adabiyotshunoslik, chog'ishtirma tilshunoslik va tarjimashunoslik
10.00.04 – Yevropa, Amerika va Avstraliya xalqlari tili va adabiyoti
08.00.01 – Iqtisodiyot nazariyasi
08.00.02 – Makroiqtisodiyot
08.00.03 – Sanoat iqtisodiyoti
08.00.04 – Qishloq xo'jaligi iqtisodiyoti
08.00.05 – Xizmat ko'rsatish tarmoqlari iqtisodiyoti
08.00.06 – Ekonometrika va statistika
08.00.07 – Moliya, pul muomalasi va kredit
08.00.08 – Buxgalteriya hisobi, iqtisodiy tahlil va audit
08.00.09 – Jahon iqtisodiyoti
08.00.10 – Demografiya. Mehnat iqtisodiyoti
08.00.11 – Marketing
08.00.12 – Mintaqaviy iqtisodiyot
08.00.13 – Menejment
08.00.14 – Iqtisodiyotda axborot tizimlari va texnologiyalari
08.00.15 – Tadbirkorlik va kichik biznes iqtisodiyoti
08.00.16 – Raqamli iqtisodiyot va xalqaro raqamli integratsiya
08.00.17 – Turizm va mehmonxona faoliyati

Ma'lumot uchun, OAK Rayosatining 2024-yil 28-avgustdagi 360/5-son qarori bilan "Dissertatsiyalar asosiy ilmiy natijalarini chop etishga tavsiya etilgan milliy ilmiy nashrlar ro'yxati"ga texnika va iqtisodiyot fanlari bo'yicha "Muhandislik va iqtisodiyot" jurnali ro'yxatga kiritilgan.

Muassis: "Tadbirkor va ishbilarmon" MChJ

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1. Toshkent shahridagi G.V.Plexanov nomidagi Rossiya iqtisodiyot universiteti
2. Toshkent davlat iqtisodiyot universiteti
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9. Jizzax politexnika instituti

MUNDARIJA • СОДЕРЖАНИЕ • CONTENTS

MUNDARIJA

VELOCITY CALCULATION AND ANALYSIS OF MANIPULATOR MOTION BASED ON THE DIFFERENTIAL APPROACH

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Annotatsiya. Manipulyator bo‘g‘inlari tezliklarini hisoblashda differensial kinematik yondashuvning qo‘llanilishi tahlil qilinadi. Jacobian matritsasi asosida ishchi organning fazoviy tezligi bilan bo‘g‘inlar tezliklari o‘rtasidagi lokal bog‘liqlik aniqlanadi. Resolved-rate velocity modeli shakllantirilib, teskari akslantirishning uchta amaliy usuli — Jacobian matritsasini bevosita inversiyalash, Moore–Penrose psevdoinversiyasi hamda damped least-squares regulyarlashtirish usuli — qiyosiy tahlil qilinadi. Ikki bo‘g‘inli tekislik manipulyatori misolida manipulyator konfiguratsiyasiga qarab bo‘g‘inlar tezliklari talabining o‘zgarishi hamda singulyar holatlar yaqinida regulyarlashtirilgan yechimning ustunliklari ko‘rsatib beriladi. Natijalar differensial yondashuv trajektoriyani bajarish jarayonida singulyarlik ko‘rsatkichlari, manipulabellik va bo‘g‘inlar tezligi cheklovlari doimiy nazorat qilingan sharoitda real vaqt rejimidagi manipulyator boshqaruvi uchun samarali yechim ekanligini tasdiqlaydi.

Kalit so‘zlar: differensial kinematika, Jacobian matritsasi, manipulyator tezligi, resolved-rate boshqaruvi, Moore–Penrose psevdoinversiyasi, damped least-squares usuli, singulyarlik tahlili.

Аннотация. Дифференциальный кинематический подход применяется для расчёта скоростей звеньев манипулятора. На основе матрицы Якоби устанавливается локальная зависимость между скоростью рабочего органа в рабочем пространстве и скоростями звеньев в пространстве обобщённых координат. Сформулирована модель resolved-rate velocity и выполнен сравнительный анализ трёх практических методов обратного отображения: прямого обращения матрицы Якоби, псевдообратной матрицы Мура–Пенроуза и метода регуляризации damped least-squares. На примере двухзвенного плоского манипулятора показано влияние конфигурации механизма на требуемые скорости звеньев, а также преимущества регуляризованного решения вблизи сингулярных положений. Полученные результаты подтверждают эффективность дифференциального подхода для управления манипуляторами в режиме реального времени при непрерывном контроле показателей сингулярности, манипулируемости и ограничений скоростей звеньев.

Ключевые слова: дифференциальная кинематика, матрица Якоби, скорость манипулятора, resolved-rate управление, псевдообратная матрица Мура–Пенроуза, метод damped least-squares, анализ сингулярностей.

Abstract. The differential kinematic approach is applied to calculate manipulator joint velocities by establishing the local relationship between end-effector velocity in task space and joint-rate motion in configuration space through the Jacobian matrix. The resolved-rate velocity model is formulated, and three practical inverse-mapping techniques are analyzed: direct Jacobian inversion, the Moore–Penrose pseudoinverse, and damped least-squares regularization. A planar two-link manipulator is employed as a numerical example to demonstrate how joint-speed requirements vary with manipulator configuration and to illustrate the improved robustness of regularized inversion near singular configurations. The results confirm that the differential approach provides an effective framework for real-time manipulator control when singularity measures, manipulability indices, and joint-rate limits are continuously monitored during trajectory execution.

Keywords: differential kinematics; Jacobian matrix; manipulator velocity; resolved-rate control; Moore–Penrose pseudoinverse; damped least-squares; singularity analysis.

INTRODUCTION

Velocity calculation is one of the most important operations in robot manipulation because most practical tasks are performed as continuous motions rather than as isolated point-to-point displacements. During path following, orientation maintenance, compliance tasks, and target tracking, the control system must repeatedly compute joint velocities that generate the required instantaneous motion of the end-effector. For this purpose, the differential approach is widely adopted because it linearizes manipulator kinematics at the current configuration and expresses the relationship between motions through the Jacobian matrix [1].

The significance of this method originates from resolved-rate motion control, in which a desired Cartesian velocity is directly converted into joint-space commands during each control cycle. Whitney established the foundation of this technique and demonstrated its suitability for real-time robot control [1]. Subsequent studies extended the same framework to redundant manipulators, dexterity analysis, and singularity-robust control strategies [2]–[8].

From an engineering perspective, the differential model serves a dual purpose. First, it provides a computationally efficient means of generating the next joint-velocity command. Second, it reveals the local motion capabilities of the manipulator. The same Cartesian command may require relatively small joint rates in one configuration but substantially larger joint rates in another. Consequently, velocity calculation cannot be separated from the analysis of conditioning, singularity, and dexterity. Therefore, the focus is placed not only on the derivation of differential velocity equations but also on the interpretation of their implications for manipulator control.

MAVZUGA OID ADABIYOTLAR SHARHI

Manipulyatorlar harakat tezligini differensial yondashuv asosida hisoblash bo‘yicha dastlabki muhim ilmiy ishlardan biri D. E. Whitney tomonidan amalga oshirilgan. U 1969-yilda manipulyatorning ishchi organi tezligini bo‘g‘inlar tezligiga aylantirishga asoslangan resolved motion rate control usulini taklif qilib, ushbu yondashuvning real vaqt rejimidagi boshqaruv tizimlari uchun amaliy ahamiyatini asoslab berdi [1]. Keyinchalik T. Yoshikawa manipulyatorning harakat imkoniyatlarini baholash uchun manipulabellik ko‘rsatkichini ishlab chiqdi va Jacobian matritsasining xossalari robotning lokal tezlik imkoniyatlari hamda singulyar holatlarga yaqinligini aniqlashda muhim ekanligini ko‘rsatdi [2].

Redundant manipulyatorlarda bo‘g‘inlar tezligini hisoblash masalasi C. A. Klein va C. H. Huang tomonidan psevdoinvers boshqaruv usullari asosida tadqiq etilgan. Ularning ishlari Moore–Penrose psevdoinversiyasi yordamida mumkin bo‘lgan ko‘plab yechimlar ichidan minimal normali yechimni aniqlash imkonini ko‘rsatadi [3]. B. Siciliano esa redundant robotlarning kinematik boshqaruvini umumlashtirib, null-fazo komponentidan asosiy vazifaga xalaqit bermagan holda qo‘shimcha boshqaruv maqsadlari uchun foydalanish mumkinligini asoslagan [4].

Singulyar konfiguratsiyalar yaqinida tezliklarning keskin ortishini kamaytirish bo‘yicha C. W. Wampler damped least-squares usulini taklif etib, regulyarlashtirish orqali hisoblash barqarorligini oshirish mumkinligini ko‘rsatgan [5]. S. Chiaverini task-priority boshqaruv tizimlari uchun singulyarlikka bardoshli yondashuvni rivojlantirgan [6]. A. A. Maciejewski va C. A. Klein singulyar holatlardan o‘tishda sonli filtrlash usullarini tahlil qilgan bo‘lsa [7], Y. Nakamura va H. Hanafusa teskari kinematik yechimlarning singulyarlikka nisbatan barqaror shakllarini ishlab chiqqan [8]. Mazkur tadqiqotlar differensial kinematika, Jacobian matritsasi, psevdoinversiya va regulyarlashtirish usullari manipulyator tezligini ishonchli hisoblashning asosiy ilmiy-uslubiy bazasini tashkil etishini ko‘rsatadi.

RESEARCH METHODOLOGY

Differential Kinematic Formulation

Let denote the vector of joint coordinates, and let represent the task-space variable associated with the end-effector position or pose. If the forward kinematic model is written as

$$x=f(q),$$

then differentiating it with respect to time yields the local linear relationship between Cartesian motion and joint rates:

$$\dot{x} = J(q)\dot{q}, \qquad (1)$$

where is the manipulator Jacobian matrix. Equation (1) maps joint velocities to the instantaneous Cartesian velocity. Since this mapping is local, the Jacobian must be updated continuously as the manipulator moves.

If the Jacobian is square and non-singular, the desired joint velocity can be obtained by direct inversion:

$$\dot{q} = J^{-1}(q)\dot{x} \qquad (2)$$

This approach is simple and efficient; however, it becomes unreliable near singular configurations because the inverse tends to amplify small changes in task-space commands into large joint-rate demands.

For redundant manipulators, where , infinitely many joint-velocity solutions may exist. In that case, the minimum-norm solution is obtained through the Moore–Penrose pseudoinverse:

$$\dot{q} = J^{\dagger}\dot{x} + \left(I - J^{\dagger}J\right)z \qquad (3)$$

where is an arbitrary null-space vector. This null-space term can be used to incorporate secondary objectives, such as obstacle avoidance, posture optimization, or joint-limit avoidance, without altering the primary Cartesian task [3], [4], [6].

When exact inversion leads to excessive joint speeds near singular configurations, a damped least-squares formulation is commonly adopted:

$$\dot{q} = J^{T}(JJ^{T} + \lambda^{2}I)^{-1}\dot{x} \qquad (4)$$

This method improves numerical robustness by reducing the sensitivity of the inverse solution at the expense of introducing a small Cartesian tracking error [5], [8].

Dexterity and Singularity Measures

The Jacobian provides more than a velocity mapping. Its singular values characterize how efficiently the manipulator can generate motion in different Cartesian directions. When the smallest singular value becomes very small, the manipulator approaches a singular configuration, and even a modest Cartesian velocity may require very large joint rates.

A widely used scalar dexterity indicator is the manipulability index introduced by Yoshikawa [2]:

$$w(q) = \sqrt{det(JJ^{T})} \qquad (5)$$

A large value of indicates good local dexterity, whereas corresponds to a singular configuration. Another practical indicator is the condition number:

$$\kappa = \frac{\sigma_{max}}{\sigma_{min}}$$

where and are the largest and smallest singular values of the Jacobian, respectively. A large condition number indicates poor conditioning and a strong directional imbalance in motion capability.

Numerical Example and Evaluation Procedure

To illustrate the analysis, a planar two-link revolute manipulator with link lengths $l_1 = l_2 = 1$ mis considered. The end-effector position is given by

$$x = l_1 \cos q_1 + l_2 \cos(q_1 + q_2),$$
$$y = l_1 \sin q_1 + l_2 \sin(q_1 + q_2)$$

The corresponding Jacobian matrix becomes

$$J(q) = \begin{bmatrix} -l_1 \sin q_1 - l_2 \sin(q_1 + q_2) & -l_2 \sin(q_1 + q_2) \\ l_1 \cos q_1 + l_2 \cos(q_1 + q_2) & l_2 \cos(q_1 + q_2) \end{bmatrix} \qquad (6)$$

The determinant of this Jacobian is

$$det(J) = l_1 l_2 \sin q_2 \qquad (7)$$

This result shows that the manipulator becomes singular when or . These configurations correspond to fully extended or fully folded manipulator postures, in which one instantaneous Cartesian direction of motion is lost.

For the numerical test, is fixed, and the desired task-space velocity is chosen as

$$\dot{x} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix} m/s \qquad (8)$$

Joint rates are computed using direct inversion and the damped least-squares method with . The objective is to observe how the required joint-speed norm changes as approaches a singular configuration.

ANALYSIS AND RESULTS

The numerical calculations show that the magnitude of the required joint velocity increases significantly as the manipulator approaches a singular configuration. This trend is summarized in Table 1.

Table 1. Joint-Speed Norm for the Planar 2R Example

q□ (deg)	Inverse (rad/s)	DLS (rad/s)
90	0.062	0.062
60	0.100	0.096
45	0.163	0.151
30	0.291	0.246
20	0.480	0.340
10	1.038	0.393
5	2.150	0.285

The results indicate that the inverse-Jacobian solution performs well in the central region of the workspace but becomes increasingly demanding as approaches zero. At , the inverse-based joint-speed norm reaches approximately **2.150 rad/s**, which is considerably larger than the value obtained in well-conditioned configurations. In contrast, the damped least-squares solution remains bounded and effectively limits the increase in joint speed near singular configurations.

The results also confirm the geometric interpretation of singularity. As becomes small, the determinant of the Jacobian approaches zero, and the manipulator gradually loses the ability to generate arbitrary Cartesian velocities without requiring excessive joint motion. Consequently, the differential approach not only computes velocity but also reveals the local motion limitations of the manipulator.

A qualitative plot of the results would show a sharp increase in for the inverse-Jacobian solution as the elbow angle approaches the singular region, whereas the damped least-squares curve remains smoother and bounded. This difference is particularly important for real-time implementation, where actuator limits and numerical stability must be respected.

The obtained results demonstrate that manipulator velocity calculation is not merely an algebraic transformation but also a direct measure of motion feasibility and control robustness. Direct inversion is particularly effective when the Jacobian is well-conditioned because it accurately reproduces the desired Cartesian velocity. However, as the manipulator approaches a singular configuration, the same method may require very large joint rates and amplify the effects of sensor noise or model uncertainty. This observation explains why singularity-robust inverse schemes have become standard in practical manipulator control [5]–[8].

For non-redundant manipulators, the primary objective is often to identify poorly conditioned regions of the workspace and modify the planned path or orientation accordingly. For redundant manipulators, the problem becomes more flexible because the null space can be utilized constructively. The controller can preserve dexterity, avoid obstacles, reduce energy consumption, or maintain favorable joint configurations while continuing to satisfy the primary Cartesian task [3], [4], [6].

The study also highlights the value of monitoring dexterity indicators, such as manipulability and the condition number. These metrics provide an online estimate of whether a given configuration is suitable for the commanded motion. In high-speed robotic systems, such indicators can be integrated into supervisory control logic to switch between inverse strategies, activate damping, or trigger path replanning before actuator saturation occurs.

Another important consideration is the local nature of the differential model. **The Jacobian** provides an accurate instantaneous approximation; however, it does not guarantee that the manipulator will remain far from singularities or joint limits over an extended motion interval. For this reason, differential control is most effective when combined with trajectory planning and higher-level motion coordination.

Overall, the differential approach remains one of the most versatile tools in robot motion analysis. It provides a compact mathematical framework, supports real-time implementation, and yields interpretable measures of local motion quality. These properties make it especially useful in industrial robotics, service manipulation, and research on redundant and adaptive robotic systems.

CONCLUSION AND SUGGESTIONS

The calculation and analysis of manipulator velocity based on the differential kinematic approach demonstrate that the Jacobian-based model provides a direct relationship between task-space velocity and joint rates, while the Moore–Penrose pseudoinverse and damped least-squares formulations extend the applicability of the method to redundant and singularity-prone configurations. The planar two-link example confirms that joint-speed demand depends strongly on the manipulator configuration and increases rapidly as the manipulator approaches singularity.

The main conclusion is that reliable manipulator velocity control should combine Jacobian-based computation with the continuous analysis of singular values, manipulability, and the condition number. Under these conditions, the differential approach remains a practical and analytically transparent foundation for real-time manipulator motion generation and control.

To improve the robustness and reliability of manipulator motion control, it is recommended to continuously monitor singularity measures, manipulability, and joint-rate limits during trajectory execution. In addition, selecting an appropriate inverse solution, including damped least-squares regularization when operating near singular configurations, can contribute to improved numerical stability and more reliable real-time control performance.

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