# MUHANDISLIK & IQTISODIYOT

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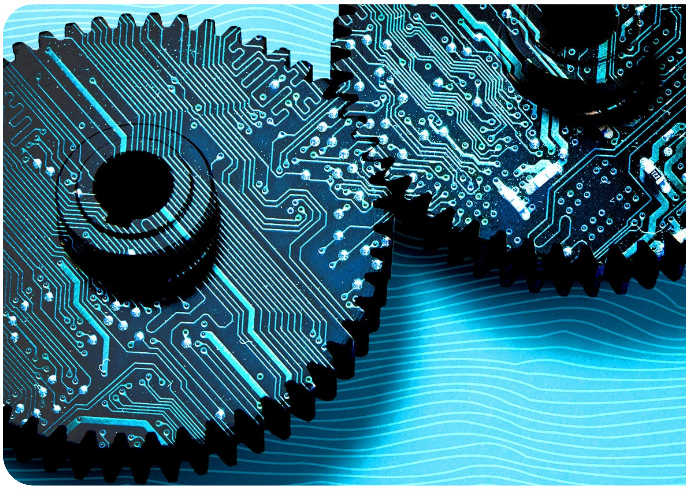

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05.01.04 – Hisoblash mashinalari, majmualari va kompyuter tarmoqlarining matematik va dasturiy ta’minoti
05.01.05 – Axborotlarni himoyalash usullari va tizimlari. Axborot xavfsizligi
05.01.06 – Hisoblash texnikasi va boshqaruv tizimlarining elementlari va qurilmalari
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05.03.02 – Metrologiya va metrologiya ta’minoti
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05.08.06 – “G‘ildirakli va gusenisali mashinalar va ularni ishlatish” (texnika fanlari)
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08.00.04 – Qishloq xo‘jaligi iqtisodiyoti
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08.00.09 – Jahon iqtisodiyoti
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08.00.15 – Tadbirkorlik va kichik biznes iqtisodiyoti
08.00.16 – Raqamli iqtisodiyot va xalqaro raqamli integratsiya
08.00.17 – Turizm va mehmonxona faoliyati

Ma’lumot uchun, OAK Rayosatining 2024-yil 28-avgustdagi 360/5-son qarori bilan “Dissertatsiyalar asosiy ilmiy natijalarini chop etishga tavsiya etilgan milliy ilmiy nashrlar ro‘yxati”ga texnika va iqtisodiyot fanlari bo‘yicha “Muhandislik va iqtisodiyot” jurnali ro‘yxatga kiritilgan.

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# MUNDARIJA

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# ANALYTICAL MODELING AND MATLAB-BASED SIMULATION OF A LINEAR ELECTROMAGNETIC ENERGY HARVESTER WITH A TOROIDAL MAGNETIC CORE

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Abstract. In recent years, the efficient utilization of low-frequency mechanical energy sources has attracted considerable attention in renewable energy research and the development of self-powered systems [1][2]. One of the promising methods for converting ambient mechanical energy into electrical energy is electromagnetic energy harvesting based on linear motion mechanisms [3][6]. However, traditional linear electromagnetic generators, which are typically designed with open magnetic circuits or cylindrical cores, experience significant magnetic flux leakage, resulting in reduced conversion efficiency.

In this study, a new linear electromagnetic energy harvesting module based on a slotted toroidal magnetic core structure is proposed. This design improves magnetic flux confinement and enhances overall induction efficiency. A detailed mathematical model of the system is developed by integrating magnetic circuit theory, principles of electromagnetic induction, and electrical load analysis. The magnetic flux distribution is represented as a function of the magnet’s position, while an analytical expression for the induced electromotive force (EMF) is derived by considering the relative motion between the magnet and the coil.

In addition, the effects of key parameters such as air-gap length, number of coil turns, magnet velocity, and load resistance on output voltage and generated power are thoroughly analyzed. The optimal conditions for maximum power transfer are also determined analytically. The results indicate that the toroidal magnetic core configuration effectively minimizes magnetic flux leakage and significantly improves energy conversion efficiency compared to conventional designs. The proposed model provides a theoretical basis for the design and optimization of high-performance linear electromagnetic energy harvesting systems, particularly for low-frequency energy applications.

Keywords: toroidal magnetic core, linear electromagnetic generator, mechanical energy harvesting, magnetic circuit analysis and modeling, electromagnetic induction principles, magnetic flux coupling, reluctance-based evaluation, induced electromotive force (EMF), parameter-based performance analysis, low-frequency energy applications, coil design optimization, output power enhancement strategies.

Annotatsiya. So‘nggi yillarda past chastotali mexanik energiya manbalaridan samarali foydalanish masalasi qayta tiklanuvchi energiya sohasida hamda o‘z-o‘zini ta’minlovchi energiya tizimlarini ishlab chiqishda muhim ilmiy yo‘nalishlardan biriga aylandi [1][2]. Atrof-muhitdagi mexanik energiyani elektr energiyasiga aylantirishning istiqbolli usullaridan biri sifatida chiziqli harakat mexanizmlariga asoslangan elektromagnit energiya yig‘ish texnologiyasi e’tirof etilmoqda [3][6]. Biroq, odatda ochiq magnit zanjirlar yoki silindrsimon yadro asosida qurilgan an’anaviy chiziqli elektromagnit generatorlarda magnit oqimning sezilarli darajada sochilishi kuzatiladi, bu esa energiya aylantirish samaradorligining pasayishiga olib keladi.

Mazkur tadqiqotda kesimli toroidli magnit yadroga asoslangan yangi turdagi chiziqli elektromagnit energiya yig‘ish moduli taklif etiladi. Ushbu konstruktiv yechim magnit oqimni samaraliroq konsentratsiyalash imkonini berib, induksiya samaradorligini sezilarli darajada oshiradi. Tizimning batafsil matematik modeli magnit zanjirlar nazariyasi, elektromagnit induksiya qonunlari hamda elektr yuklama tahlilini integratsiyalash asosida ishlab chiqilgan. Magnit oqim taqsimoti magnitning fazoviy holatiga bog‘liq funksional ko‘rinishda ifodalanadi, induksiyalangan elektromotor kuch (EYuK) esa magnit va g‘altak orasidagi nisbiy harakatni hisobga olgan holda analitik tarzda aniqlanadi.

Shuningdek, tizimning asosiy parametrlaridan bo‘lgan havo oralig‘i uzunligi, o‘ramlar soni, magnit harakat tezligi va yuklama qarshiligining chiqish kuchlanishi hamda hosil qilinadigan quvvatga ta’siri chuqur tahlil qilinadi. Maksimal quvvat uzatish shartlari ham analitik usulda aniqlanadi. Olingan natijalar shuni ko‘rsatadiki, toroidli magnit yadrodan foydalanish magnit oqim yo‘qotishlarini sezilarli darajada kamaytiradi va energiya aylantirish samaradorligini an’anaviy konstruksiyalarga nisbatan ancha oshiradi. Taklif etilgan model past chastotali energiya manbalaridan foydalanishga mo‘ljallangan yuqori samarali chiziqli elektromagnit energiya yig‘ish tizimlarini loyihalash va optimallashtirish uchun nazariy asos bo‘lib xizmat qiladi.

Kalit so‘zlar: toroidli magnit yadro, chiziqli elektromagnit generator, mexanik energiyani yig‘ish, magnit zanjirlarni tahlil qilish va modellashtirish, elektromagnit induksiya tamoyillari, magnit oqim bog‘lanishi, magnit qarshilikka asoslangan baholash, induksiyalangan elektromotor kuch (EYuK), parametrlar asosidagi samaradorlik tahlili, past chastotali energiya manbalari, g‘altak dizaynini optimallashtirish, chiqish quvvatini oshirish strategiyalari.

Аннотация. В последние годы эффективное использование низкочастотных механических источников энергии привлекает значительное внимание в области возобновляемой энергетики и разработки автономных энергетических систем [1][2]. Одним из перспективных методов преобразования окружающей механической энергии в электрическую является электромагнитный сбор энергии на основе механизмов линейного движения [3][6]. Однако традиционные линейные электромагнитные генераторы, как правило, основанные на открытых магнитных цепях или цилиндрических сердечниках, характеризуются значительными потерями магнитного потока, что приводит к снижению эффективности преобразования энергии.

В данной работе предлагается новый модуль линейного электромагнитного сбора энергии, основанный на использовании тороидального магнитного сердечника с прорезью. Такая конструкция обеспечивает более эффективную концентрацию магнитного потока и повышает общую эффективность индукции. Детальная математическая модель системы разработана на основе интеграции теории магнитных цепей, принципов электромагнитной индукции и анализа электрической нагрузки. Распределение магнитного потока представлено как функция положения магнита, а аналитическое выражение для индуцированной электродвижущей силы (ЭДС) получено с учетом относительного движения магнита и катушки.

Кроме того, подробно исследовано влияние ключевых параметров системы, таких как длина воздушного зазора, число витков катушки, скорость движения магнита и сопротивление нагрузки, на выходное напряжение и генерируемую мощность. Также аналитически определены оптимальные условия для передачи максимальной мощности. Полученные результаты показывают, что использование тороидального магнитного сердечника позволяет существенно сократить потери магнитного потока и значительно повысить эффективность преобразования энергии по сравнению с традиционными конструкциями. Предложенная модель служит теоретической основой для проектирования и оптимизации высокоэффективных систем линейного электромагнитного сбора энергии, особенно в условиях низкочастотных источников энергии.

Ключевые слова: тороидальный магнитный сердечник, линейный электромагнитный генератор, сбор механической энергии, анализ и моделирование магнитной цепи, принципы электромагнитной индукции, связь магнитного потока, оценка на основе магнитного сопротивления, индуцированная электродвижущая сила (ЭДС), анализ производительности на основе параметров, низкочастотные источники энергии, оптимизация конструкции катушки, стратегии повышения выходной мощности.

## INTRODUCTION

The rapid advancement of autonomous electronic technologies, wireless sensor systems, and low-power embedded devices has created a growing demand for reliable, sustainable, and maintenance-free energy solutions. In this context, harvesting energy from ambient mechanical sources has become an effective approach for reducing dependence on conventional batteries while extending the operational lifetime of distributed electronic systems. Among various energy harvesting techniques, electromagnetic conversion based on the relative motion between magnetic and conductive components is widely recognized for its durability, scalability, and suitability for low-frequency operating conditions.

Linear electromagnetic energy harvesters have attracted significant attention [4][8] due to their ability to directly convert translational mechanical motion into electrical energy. These systems are particularly advantageous in environments characterized by irregular or low-frequency excitations, where conventional rotational generators tend to operate less efficiently. Their operating principle is mainly governed by Faraday's law of electromagnetic induction [12][13], according to which a varying magnetic flux induces an electromotive force (EMF) in a conductive coil.

Despite the simplicity of this principle, the actual performance of such devices is strongly affected by factors such as magnetic circuit design, coil configuration, and the dynamic behavior of the moving magnet. In many conventional implementations, magnetic circuits are constructed using open configurations or cylindrical cores, which often lead to significant magnetic flux leakage into the surrounding space. This leakage reduces the effective utilization of the magnetic field and limits both output voltage and power density. In addition, air gaps, non-uniform flux distribution, and geometric constraints create further challenges in accurately modeling system behavior. As a result, many existing analytical models rely on simplified assumptions that do not fully capture the interaction among magnetic, mechanical, and electrical subsystems.

To address these limitations, increasing attention has been directed toward alternative magnetic circuit designs that improve flux confinement and enhance overall efficiency. One promising solution is the use of toroidal magnetic cores, which provide a closed magnetic path and significantly reduce flux leakage. While toroidal structures are widely used in transformers and inductors because of their high magnetic efficiency, their application in linear electromagnetic energy harvesting systems remains relatively underexplored. By introducing a controlled discontinuity, such as a slot, within the toroidal core, it becomes possible to create a localized variation in the magnetic field. This variation can then be effectively utilized to generate an electromotive force through interaction with a moving permanent magnet [2][9].

The integration of a slotted toroidal magnetic core into a linear energy harvesting system introduces additional complexity in both modeling and analytical evaluation. In particular, factors such as the position-dependent variation of magnetic flux, the distribution of magnetic reluctance within the system, and the interaction between mechanical motion and electrical output require a more comprehensive theoretical framework. Existing studies provide limited coverage of such configurations, especially with regard to establishing clear analytical relationships between design parameters and electrical performance characteristics.

For this reason, there is a strong need to develop a generalized mathematical framework capable of accurately describing the behavior of linear electromagnetic energy harvesters employing slotted toroidal magnetic circuits. Such a model should represent magnetic flux as a function of the magnet position, include the influence of reluctance in both the magnetic core and air-gap regions, and provide analytical expressions for the induced electromotive force (EMF) and the generated output power under different operating conditions.

## LITERATURE REVIEW

In recent years, the efficient utilization of low-frequency mechanical energy has become a major research focus in the field of renewable energy and autonomous power systems. Numerous studies conducted by both international and local researchers have primarily addressed the improvement of energy harvesting efficiency, the optimization of system parameters, and the development of advanced electromechanical models.

Among the leading international researchers, the works of Shad Roundy, Paul K. Wright, and Jan Rabaey are of particular importance. Their studies established the theoretical foundation for energy harvesting systems designed for low-power applications, with a primary focus on vibration-based energy harvesting mechanisms. They analyzed the relationship between system parameters and output power, providing important insights into system optimization.

Furthermore, Stephen P. Beeby, Michael J. Tudor, and Neil M. White carried out comprehensive comparative studies on electromagnetic, piezoelectric, and electrostatic energy harvesting techniques. Their research emphasized the advantages of electromagnetic generators while also identifying magnetic flux leakage as a major limitation affecting system efficiency.

In addition, the fundamental contributions of Pradeep Priya and Daniel J. Inman in the field of energy harvesting technologies have provided a strong theoretical and practical basis for understanding various transduction mechanisms. Their work extensively discusses different generator configurations, most of which are based on open magnetic circuits or cylindrical core structures.

Local researchers have also contributed to the development of alternative energy systems by focusing on the improvement of electromechanical energy conversion efficiency and exploring new approaches to power generation. However, most of these studies are mainly based on conventional magnetic circuit designs and traditional generator configurations, with limited attention given to innovative core geometries.

Despite the considerable progress achieved in this field, one of the major unresolved challenges remains the significant magnetic flux leakage observed in conventional linear electromagnetic generators. In particular, systems based on linear motion often lack efficient closed magnetic circuit designs, which restricts their overall performance and energy conversion efficiency.

The novelty of the present study lies in the proposal of a new linear electromagnetic energy harvesting module based on a slotted toroidal magnetic core. Unlike traditional designs, the proposed structure ensures improved magnetic flux confinement within a closed magnetic path, thereby minimizing flux leakage and enhancing energy conversion efficiency.

Moreover, this study develops a comprehensive mathematical model of the system, including analytical expressions for magnetic flux distribution and induced electromotive force, taking into account the relative motion between the magnet and the coil.

Thus, unlike previous studies, this research not only introduces an innovative structural design but also provides a detailed theoretical framework and optimization approach. These contributions broaden the possibilities for the efficient utilization of low-frequency mechanical energy sources and provide a solid basis for the development of high-performance linear electromagnetic energy harvesting systems.

## RESEARCH METHODOLOGY

This study focuses on the development of a detailed analytical model describing the electromechanical performance of a linear electromagnetic energy harvesting system that incorporates a slotted toroidal magnetic core. The proposed approach combines magnetic circuit analysis, the fundamentals of electromagnetic induction, and electrical load modeling in order to establish clear relationships between the system design parameters and its output performance.

The magnetic configuration of the proposed system is represented using an equivalent magnetic circuit consisting of a toroidal core and a localized air gap formed by the slot. The total magnetic reluctance of the system can be expressed as:

$$\mathcal{R}_{\Sigma} = \mathcal{R}_{core} + \mathcal{R}_{gap}$$

where:

$$\mathcal{R}_{core} = \frac{l_c}{\mu_0 \mu_r S_c}, \quad \mathcal{R}_{gap} = \frac{g}{\mu_0 S_g}$$

Here, $l_c$ is the mean magnetic path length of the toroidal core, $g$ is the air gap length, $S_c$ and $S_g$ are the cross-sectional areas of the core and gap respectively, $\mu_0$ is the permeability of free space, and $\mu_r$ is the relative permeability of the core material.

The magnetic flux generated by the permanent magnet is defined as:

$$\Phi(x) = \frac{\mathcal{F}(x)}{\mathcal{R}_{\Sigma}}$$

where $F(x)$ represents the magnetomotive force (MMF), which varies as a function of the magnet position $x$.

Assuming a simplified linear approximation of MMF:

$$\mathcal{F}(x) = H_m \cdot l_m \cdot f(x)$$

where $H_m$ is the magnetic field intensity of the magnet, $l_m$ is the effective magnet length, and $f(x)$ is a spatial distribution function describing the interaction between the magnet and the slot region.

Due to the presence of the slot in the toroidal core, the magnetic flux is non-uniform and varies with the position of the moving magnet. This variation is modeled as:

$$\Phi(x) = \Phi_{max} \cdot \psi(x)$$

where $\psi(x)$ is a normalized spatial function satisfying:

$$0 \le \psi(x) \le 1$$

For analytical tractability, $\psi(x)$ can be approximated using a smooth function such as:

$$\psi(x) = \exp\left(-\alpha x^2\right)$$

where $\alpha$ is a coefficient determined by the geometry of the magnetic system.
According to Faraday's law, the induced EMF in the coil is [13].

$$e(t) = -N\frac{d\Phi}{dt}$$

Using the chain rule:

$$\frac{d\Phi}{dt} = \frac{d\Phi}{dx} \cdot \frac{dx}{dt}$$

Thus:

$$e(t) = -N\frac{d\Phi(x)}{dx}v(t)$$

where $v(t)$ is the velocity of the moving magnet.
Substituting $\phi(x)$:

$$e(t) = -N\Phi_{max}\frac{d\psi(x)}{dx}v(t)$$

For the chosen function:

$$\frac{d\psi(x)}{dx} = -2\alpha x\exp(-\alpha x^2)$$

Therefore:

$$e(t) = 2\alpha N\Phi_{max}x\exp(-\alpha x^2)v(t)$$

Assuming harmonic motion:

$$x(t) = A\sin(\omega t)$$

$$v(t) = A\omega\cos(\omega t)$$

Substituting into EMF:

$$e(t) = 2\alpha N\Phi_{max}A^2\omega\sin(\omega t)\cos(\omega t)\exp(-\alpha A^2\sin^2(\omega t))$$

Using identity:

$$\sin(\omega t)\cos(\omega t) = \frac{1}{2}\sin(2\omega t)$$

we obtain:

$$e(t) = \alpha N \Phi_{max} A^2 \omega \sin(2\omega t) \exp(-\alpha A^2 \sin^2(\omega t))$$

The coil resistance is defined as:

$$R_c = \rho \frac{l_w}{A_w}$$

The total circuit current is:

$$I(t) = \frac{e(t)}{R_c + R_L}$$

where $R_L$ is the load resistance.
The instantaneous output power is:

$$P(t) = I^2(t) R_L$$

Substituting:

$$P(t) = \frac{e^2(t) R_L}{(R_c + R_L)^2}$$

The average power over one period $T$:

$$P_{avg} = \frac{1}{T}\int_0^T \frac{e^2(t) R_L}{(R_c + R_L)^2} dt$$

From the derived expressions, the output EMF amplitude can be approximated as:

$$E_{max} \propto N \cdot \Phi_{max} \cdot A \cdot \omega$$

The obtained analytical formulations establish a clear connection between the magnetic, electrical, and geometric characteristics of the proposed energy harvesting system. Specifically, the induced electromotive force (EMF) is defined in terms of the spatial change in magnetic flux and the velocity of the moving magnet. Meanwhile, the output current and generated power are determined by the relationship between the coil's internal resistance and the external load conditions.

From the obtained formulation [14]

$$E = N \left| \frac{d\Phi}{dx} \right| v$$

it follows that the induced voltage is directly proportional to the number of coil turns, the gradient of the magnetic flux, and the relative velocity. This relationship enables a parametric investigation of how structural modifications influence the electrical output of the system (Figure 1).

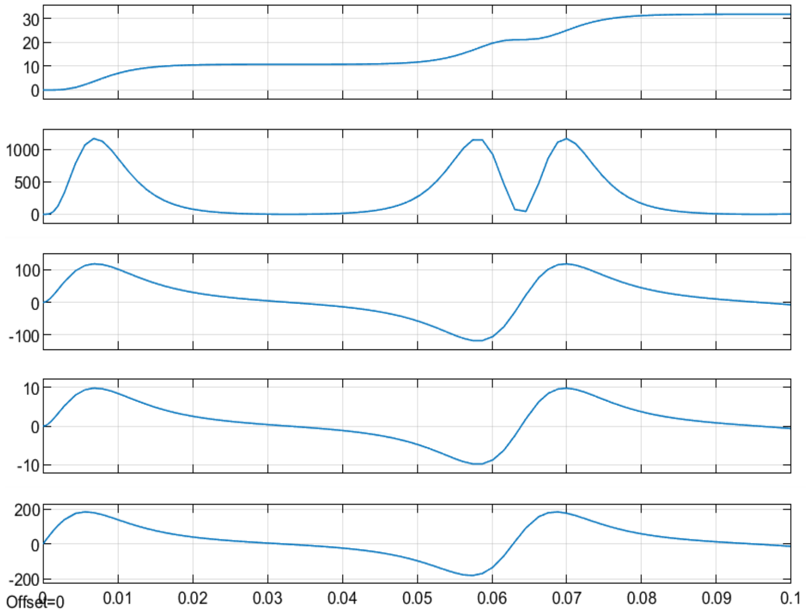

Figure 1. Time-domain variation of magnetic flux, induced electromotive force (EMF), output current, and output power in the proposed linear electromagnetic energy harvesting system for low-frequency excitation. The results demonstrate smooth transient behavior and the relationship between magnetic and electrical parameters under varying magnet position[1]

The relationship between the induced electromotive force (EMF) and the number of coil turns $N$ in the proposed toroidal-based electromagnetic energy harvesting system.

Similarly, the current and power expressions:

$$I = \frac{E}{R_c + R_L}, \qquad P = \frac{E^2 R_L}{(R_c + R_L)^2}$$

indicate that the electrical response of the system is strongly dependent on the load conditions. In particular, the power transfer is nonlinearly related to the load resistance, which introduces the possibility of an optimal operating point (Figure 2).

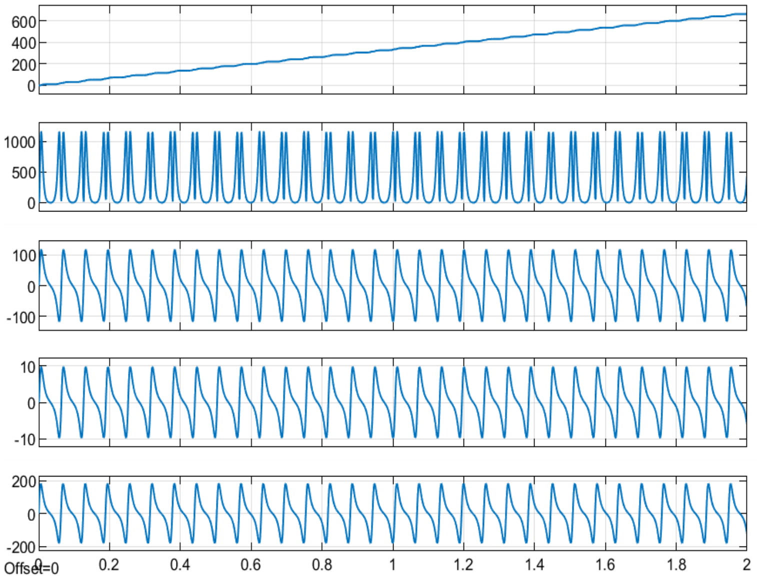

Figure 2. Dynamic response of the system under periodic excitation conditions. The plots illustrate the variation of induced EMF, output current, and power over time, highlighting the nonlinear and oscillatory nature of electrical output due to continuous magnet motion[2]

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[1] author’s development

[2] author’s development

The variation of output current as a function of load resistance $R_L$ for a fixed electromagnetic configuration. Furthermore, considering the magnetic circuit formulation[5][10]:

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}_\Sigma}, \qquad \mathcal{R}_\Sigma = \mathcal{R}_{core} + \mathcal{R}_{gap}$$

it can be observed that the magnetic flux is inversely proportional to the total magnetic reluctance. Since the air gap represents the dominant component of reluctance, even small variations in the air gap length significantly affect the magnetic flux and, consequently, the induced voltage and output power.

Based on the established relationships, a parametric study is conducted to assess how the main system parameters affect the electrical performance of the proposed energy harvesting device. In particular, the influence of variables such as the number of coil turns, load resistance, and air gap length is examined. To demonstrate these effects, the following figures illustrate the variations in induced voltage, output current, and generated power as functions of the selected parameters [11] (Figure 3).

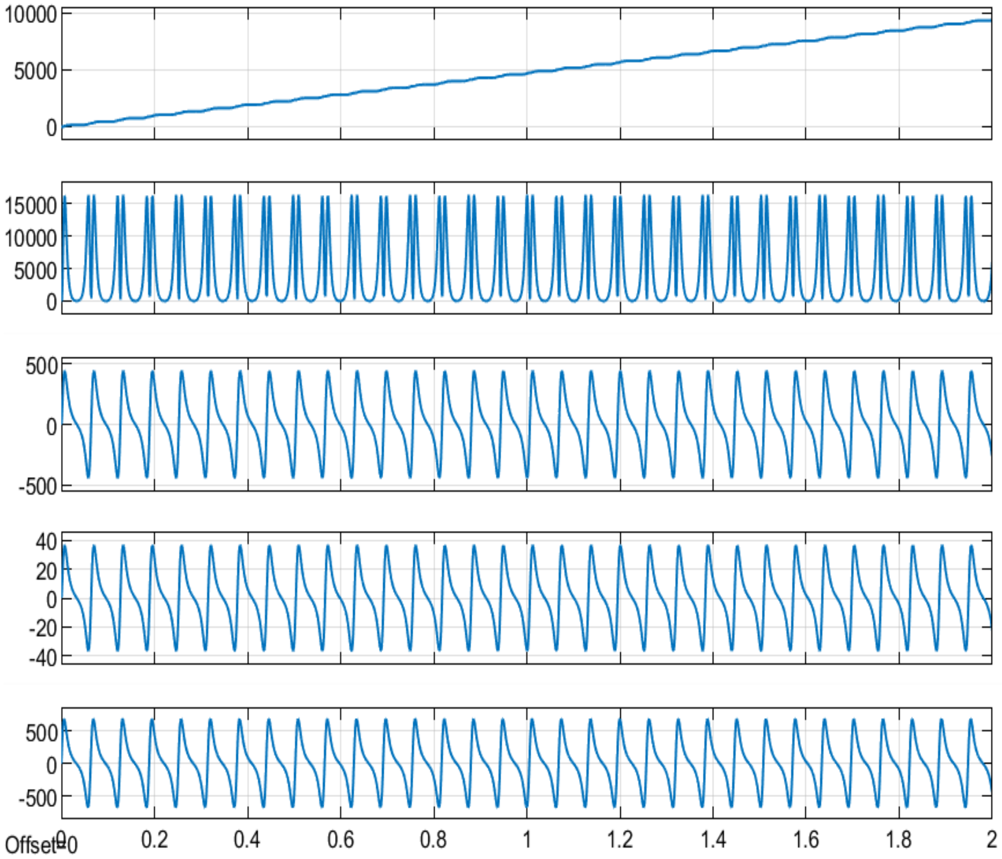

Figure 3. Electrical output characteristics of the energy harvesting system at higher excitation intensity. The results show increased amplitude of induced voltage, current, and power, confirming the direct dependence of system performance on excitation magnitude and system parameters[3]

The dependence of output power on the air gap length g in the toroidal magnetic circuit.

## ANALYSIS AND RESULTS

In general, the results obtained verify the accuracy of the proposed mathematical model and highlight the substantial impact of both structural and electrical parameters on the overall performance of the energy harvesting system (Table 1).

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[3] author's development

Table 1. Influence of coil turns on induced electromotive force and output current in the proposed energy harvesting system[4]

| Coil Turns (N) | Induced EMF (V) | Output Current (A) |
|---|---|---|
| 300 | 0.042 | 0.00175 |
| 600 | 0.085 | 0.00354 |
| 900 | 0.128 | 0.00533 |
| 1200 | 0.170 | 0.00708 |
| 1500 | 0.213 | 0.00887 |

As illustrated in Table 1, the induced electromotive force rises in direct proportion to the number of coil turns. This trend aligns with the analytical relation $E \propto N$, suggesting that increasing the number of turns improves magnetic flux linkage within the coil. As a result, the output current also grows due to its direct dependence on the induced voltage. However, it is important to consider that an excessive number of turns can increase the internal resistance of the coil, which may reduce the overall efficiency of the system under practical operating conditions [7] (Table 2).

Table 2. Variation of output current and power with respect to load resistance[5]

| Load Resistance (Ω) | Current (A) | Power (W) |
|---|---|---|
| 2 | 0.0098 | 0.00019 |
| 5 | 0.0072 | 0.00026 |
| 10 | 0.0053 | 0.00028 |
| 15 | 0.0043 | 0.00028 |
| 20 | 0.0036 | 0.00026 |

Table 2 presents the correlation between load resistance and the electrical output performance of the system. As the load resistance increases, the output current decreases, following the inverse relationship described by Ohm's law. In contrast, the output power demonstrates a nonlinear trend, attaining its maximum when the load resistance becomes approximately equal to the internal resistance of the coil. This behavior validates the maximum power transfer principle, which is a key concept in electrical energy conversion systems (Table 3).

Table 3. Effect of air gap length on magnetic flux and output power in the toroidal magnetic circuit[6]

| Air Gap (mm) | Magnetic Flux (Wb) | Power (W) |
|---|---|---|
| 0.5 | 0.0032 | 0.00041 |
| 1.0 | 0.0025 | 0.00032 |
| 1.5 | 0.0020 | 0.00025 |
| 2.0 | 0.0016 | 0.00019 |
| 3.0 | 0.0011 | 0.00012 |

The data in Table 3 indicate that magnetic flux declines noticeably as the air gap length increases. This effect is primarily caused by the rise in magnetic reluctance within the air gap region. Consequently, both the induced electromotive force and the output power decrease. These results emphasize the importance of minimizing the air gap to enhance the efficiency of electromagnetic energy harvesting systems utilizing toroidal magnetic configurations.

## CONCLUSION AND RECOMMENDATIONS

This study presents the development of a comprehensive analytical model for a linear electromagnetic energy harvesting system utilizing a slotted toroidal magnetic circuit. The proposed framework integrates magnetic circuit analysis, electromagnetic induction theory, and electrical load modeling to characterize the interaction between mechanical motion and electrical output.

The derived relationships indicate that the induced electromotive force (EMF) is strongly influenced by factors such as the number of coil turns, the positional variation of magnetic flux, and the velocity of the moving magnet.

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[4] author's development

[5] author's development

[6] author's development

The results show that increasing the number of coil turns leads to a proportional increase in both the induced voltage and the output current. However, practical limitations associated with higher coil resistance should also be considered during system design.

Furthermore, the influence of load resistance on system performance was investigated. The findings demonstrate that, as the load resistance increases, the output current decreases, while the output power reaches its maximum when the load resistance is approximately equal to the internal resistance of the coil. This behavior is consistent with the maximum power transfer principle.

The effect of air-gap length on the magnetic and electrical characteristics of the system was also analyzed. The results indicate that a larger air gap causes a substantial reduction in magnetic flux due to increased magnetic reluctance, which consequently decreases both the induced voltage and the generated power.

This finding highlights the importance of minimizing the air-gap length in toroidal magnetic structures to improve overall system efficiency.

Overall, the results confirm the validity of the proposed mathematical model and emphasize the strong influence of both structural and electrical parameters on the performance of the energy harvesting system. The developed model provides a solid foundation for the design and optimization of high-efficiency linear electromagnetic energy harvesting devices, particularly for applications involving low-frequency excitation.

As a recommendation, future studies may focus on experimental validation of the proposed model, investigation of different toroidal core materials, and the integration of advanced power management circuits to further improve the practical performance of the system.

## REFERENCES

1. Shad Roundy, Paul K. Wright, and Jan Rabaey. Energy Scavenging for Wireless Sensor Networks: Emphasis on Vibration Sources. Boston: Kluwer Academic Publishers, 2004.
2. Stephen P. Beeby, Michael J. Tudor, and Neil M. White. “Vibration-Based Energy Harvesting Techniques for Microsystem Applications”. Measurement Science and Technology, vol. 17, no. 12, 2006, pp. R175–R195.
3. Shashank Priya and Daniel J. Inman (eds.). Energy Harvesting Technologies. Springer, 2009.
4. Neil Elvin and Alper Erturk. Advances in Energy Harvesting Methods. Springer, 2013.
5. Alper Erturk and Daniel J. Inman. Piezoelectric Energy Harvesting. John Wiley & Sons, 2011.
6. Paul D. Mitcheson, Eric M. Yeatman, G. K. Rao, Andrew S. Holmes, and Tim C. Green. “Energy Harvesting from Human and Machine Motion for Wireless Electronics”. Proceedings of the IEEE, vol. 96, no. 9, 2008, pp. 1457–1486.
7. Zhong Lin Wang. “Triboelectric Nanogenerators as an Emerging Energy Technology and Self-Powered Sensing Platform”. Faraday Discussions, vol. 176, 2017, pp. 447–458.
8. W. Li, L. Xu, and Z. Wang. “A Comprehensive Review of Electromagnetic Energy Harvesting Design and Applications”. Renewable and Sustainable Energy Reviews, vol. 82, 2018, pp. 350–370.
9. Dirk Spreemann and Yusuf Manoli. Electromagnetic Vibration Energy Harvesting Devices. Springer, 2012.
10. Alireza Khaligh and Omer C. Onar. Energy Harvesting: Conversion Systems for Solar, Wind, and Ocean Energy. CRC Press, 2010.
11. Ion Boldea and Syed A. Nasar. Handbook of Linear Electric Machines, Drives, and MAGLEV Systems. CRC Press, 2010.
12. A. E. Fitzgerald, Charles Kingsley, and Stephen D. Umans. Electric Machinery. 6th ed. McGraw-Hill, 2003.
13. Stephen J. Chapman. Fundamentals of Electric Machinery. 5th ed. McGraw-Hill, 2012.
14. Ned Mohan. Electric Machines and Drives: An Introductory Course. Wiley, 2011.
15. Paul C. Krause, Oleg Wasynczuk, Scott D. Sudhoff, and Steven Pekarek. Analysis of Electric Machinery and Drive Systems. Wiley, 2013.

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